Support Vector Machine (with Python)

Tutorial 3 Yang

Outlines

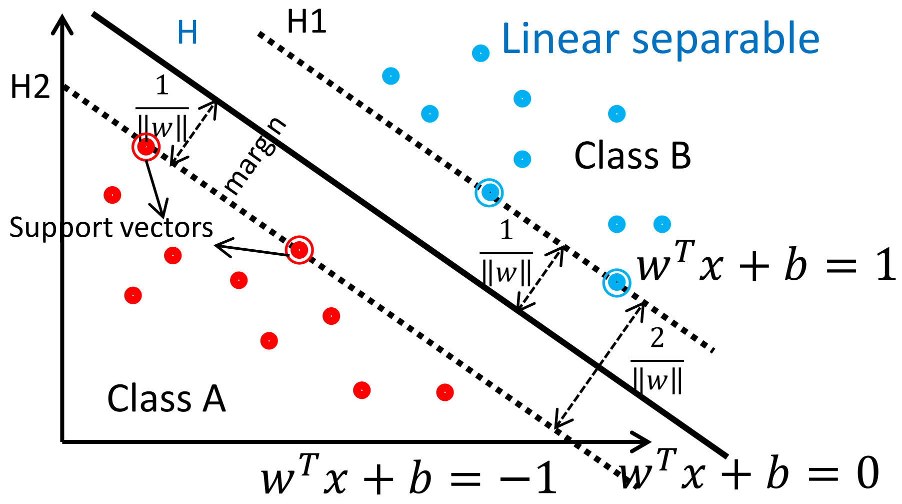
Through this tutorial, you will better know:

- What is Support Vector Machine
- The SVM in Scikit-learn – C-Support Vector Classification
- The method to train the SVM – SMO algorithm
- The parameters in SVC
- How to use the Sickit-learn.SVM
- Other SVMs in Scikit-learn

Linear model

Support vector machine:

- Margin: the smallest distance between the decision boundary and any of the samples
- maximizing the margin $\Rightarrow$ a particular decision boundary
- Location of boundary is determined by support vectors



- Canonical representation:

$$\arg\min \frac{1}{2}\|w\|^2,$$

$$s.t.\ \ t_n(w * x_i + b) \geq 1,\ \ n = 1,2,\dots,N$$

- By Lagrangian, its dual form (QP problem)

$$\min_{\vec{a}} \psi(\vec{a}) = \min_{\vec{a}} \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} t_n t_m (x_n \cdot x_m)\, a_n a_m - \sum_{n=1}^{N} a_n\,,$$

$$s.t.\, a_n \geq 0,\ \ n = 1,2,\dots,N,$$

$$\sum_{n=1}^{N} a_n t_n = 0.$$

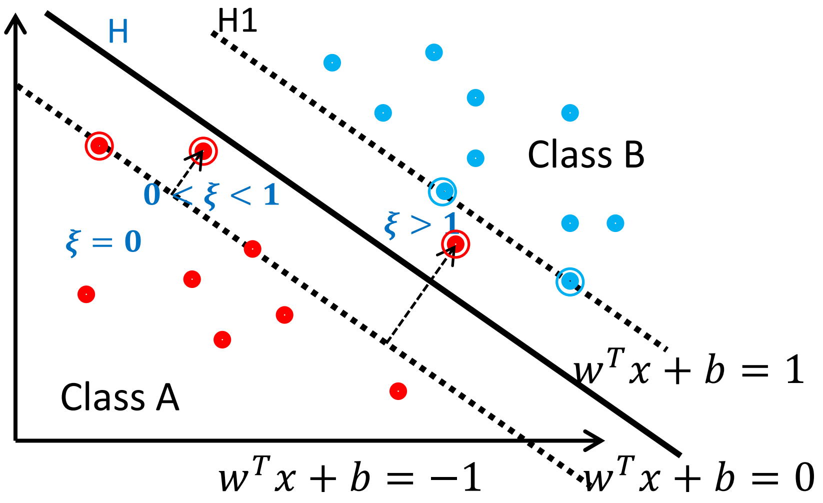
Nonlinear model

Soft margin:

- Slack variables $\xi_n \geq 0, n = 1, \ldots, N$
- Maximize the margin while softly penalizing incorrect points

$$\arg\min \frac{1}{2}\|w\|^2 + C\sum_{n=1}^{N}\xi_n,$$

$$s.t.\ \ t_n(w * x_i + b) \geq 1 - \xi_n,\ \ n = 1, \ldots, N.$$



- The corresponding dual form by Lagrangian:

$$\min_{\vec{\alpha}} \psi(\vec{a}) = \min_{\vec{\alpha}} \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} t_n t_m k(x_n, x_m)\, a_n a_m - \sum_{n=1}^{N} a_n\,,$$

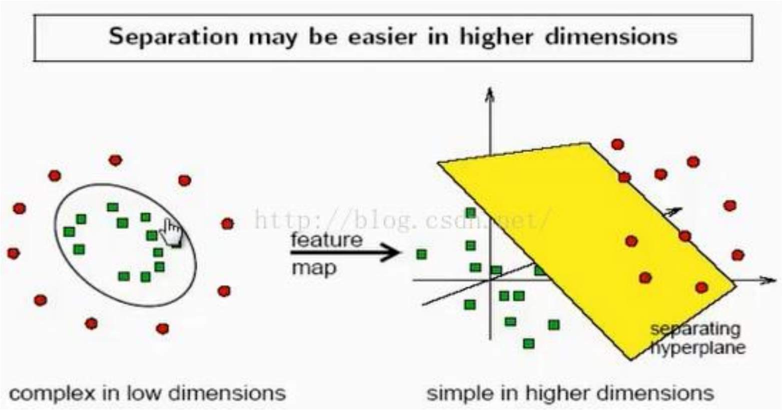
$$s.t. 0 \leq a_n \leq C,\ \ \ n = 1,2, \ldots, N,$$

$$\sum_{n=1}^{N} a_n t_n = 0.$$

C controls Trade-off between the slack variable penalty and the margin

Kernel Method

- The kernel trick (kernel substitution)
  - map the inputs into high-dimensional feature spaces properly
  - solve the problems of high complexity and computation caused by inner product



- Example: kernel function-- $k(X_i, X_j) = \langle \phi(X_i) \cdot \phi(X_j) \rangle$

Defined two vectors: $x = (x_1, x_2, x_3); y = (y_1, y_2, y_3)$

Defined the equations: $f(x) = (x_1x_1, x_1x_2, x_1x_3, x_2x_1, x_2x_2, x_2x_3, x_3x_1, x_3x_2, x_3x_3)$,

$$K(x, y) = (< x, y >)^2,$$

Assume $x = (1,2,3), y = (4,5,6)$

$f(x) = (1,2,3,2,4,6,3,6,9), f(y) = (16,20,24,20,25,36,24,30,36)$,

$< f(x), f(y) >= 16 + 40 + 72 + 40 + 100 + 180 + 72 + 180 + 324 = 1024$,

$K(x, y) = (4 + 10 + 18)^2 = 1024.$ Kernel is much simpler

sklearn.svm.SVC

- C-Support Vector Classification:
  - The implementation is based on libsvm. The fit time complexity is more than quadratic with the number of samples which makes it hard to scale to dataset with more than a couple of 10000 samples.
  - The multiclass support is handled according to a one-vs-one scheme
- LibSVM:
  - LIBSVM implements the SMO algorithm for kernelized support vector machines (SVMs), supporting classification and regression.[1]

[1] Chang, Chih-Chung; Lin, Chih-Jen (2011). "LIBSVM: A library for support vector machines". *ACM Transactions on Intelligent Systems and Technology*. 2 (3).

SMO Algorithm

- Sequential Minimal Optimization[2]:
  - A Fast Algorithm for Training Support Vector Machines
  - Quickly solve the SVM quadratic programming (QP) problem
  - The main steps:

Repeat till convergence {

1. Select some pair a_i and a_j to update next (using a heuristic that tries to pick the two that will allow us to make the biggest progress towards the global maximum).

2. Reoptimize $\Psi(\vec{a})$ with respect to a_i and a_j, while holding all the other a_k's $(k \neq i, j)$ fixed.

}

[2] Platt, John. "Sequential minimal optimization: A fast algorithm for training support vector machines." (1998).

Parameters of SVC

```
class sklearn.svm. SVC (C=1.0, kernel='rbf', degree=3, gamma='auto', coef0=0.0, shrinking=True, probability=False,
tol=0.001, cache_size=200, class_weight=None, verbose=False, max_iter=-1, decision_function_shape='ovr',
random_state=None)
```
[source]

C: Penalty parameter C of the error term, controls trade-off between the penalty and the margin, default=1.0
Kernel: 'linear', 'poly', 'rbf', 'sigmoid', 'precomputed', default= 'rbf'
degree: Degree of the polynomial kernel function
gamma: Kernel coefficient('rbf', 'poly' and 'sigmoid'), gamma=auto means 1/n_features
coef0: Independent term in kernel function. It is only significant in 'poly' and 'sigmoid'.
probability: whether to enable probability estimates (true or false)
shrinking: Whether to use the shrinking heuristic

tol: Tolerance for stopping criterion
Cache_size: Specify the size of the kernel cache
class_weight: set different penalty for different data classes by the class weight values
verbose：Enable verbose output, if enabled, may not work properly in a multithreaded context
max_iter: Hard limit on iterations within solver, or -1 for no limit
decision_function_shape: for multiple classifications, ovo for one-vs-one, ovr for one-vs-rest
random_state: The seed of the pseudo random number generator to use when shuffling the data

http://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html

Kernel selection

- Linear kernel:

Choose based on the accuracy

  - Linear: $u' * v$

Mainly for linear classification, it has fewer parameters, computing fast

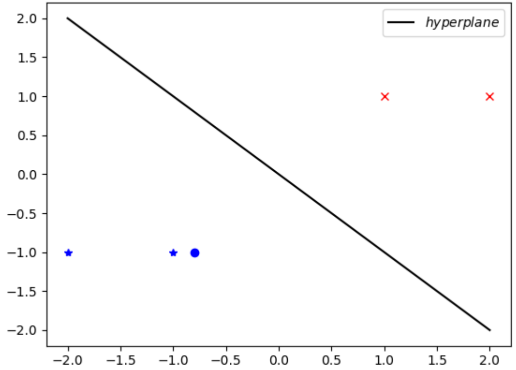
- Nonlinear kernel
  - Polynomial: $(\gamma * u' * v + coef0)^{degree}$
  - rbf: $\exp(-\gamma * |u - v|^2)$
  - Sigmoid: $\tanh(\gamma * u' * v + coef0)$

More parameters so take more time for computing, however, better performance with properly-tuned parameters

C: the penalty coefficient, low C makes the decision surface smooth, high C aims at classifying all training examples correctly
$\gamma(gamma)$: defines how far the influence of a single training example reaches, low values mean ‘far’ and high values mean ‘close’.

Example-SVC

```
from sklearn.svm import SVC
import numpy as np
X=np.array([[-1,-1],[-2,-1],[1,1],[2,1]])
y=np.array([1,1,2,2])

clf=SVC(kernel='linear')
clf.fit(X,y)
print(clf.fit(X,y))
print(clf.predict([[-0.8,-1]]))
```



import matplotlib.pyplot as plt

```
y1=y.copy()
a=np.hstack((X,y1.reshape(4,1)))
for i in range(len(a)):
    if a[i,2]==1:
        plt.plot(a[i,0],a[i,1], 'b*')
    else:
        plt.plot(a[i,0],a[i,1], 'rx')
plt.plot(-0.8, -1,'bo')
```

```
w=clf.coef_[0]          #Only for linear kernel
xx=np.linspace(-2,2)
yy=-(w[0]*xx+clf.intercept_[0])/w[1]
plt.plot(xx, yy, ‘k-’, label=‘$hyperplane$’)
plt.legend()
plt.savefig(path+’¥¥SVM.png’)
plt.show()
```

Exercise 1 -Linear model-Tasks

- First load the training data and testing data of a linear example
- Create a SVM by SVC
- Train the SVM model by the data in training file
- Classify the data in test file
- Plot the figure of data points and the hyperplane
- Pls change the parameter C and observe

Exercise 1-Linear model(1)

```
import numpy as np
import pandas as pd
from sklearn.svm import SVC
from sklearn import metrics
import matplotlib.pyplot as plt
import os

# load data
path=os.getcwd()
traindata=pd.read_csv(path+'¥¥traindata.csv')
train_x=traindata.iloc[:, :-1]
train_y=traindata.iloc[:, -1]
testdata=pd.read_csv(path+'¥¥testdata.csv')
test_x=testdata.iloc[:, :-1]
test_y=testdata.iloc[:, -1]
```

```
# introduce the SVC
clf=SVC(C=10, kernel='linear')
clf.fit(train_x, train_y)
Test_y=pd.Series(clf.predict(test_x), name='Y')

print('Classification report for classifier %s:¥n%s¥n'
    % (clf, metrics.classification_report(test_y,
Test_y)))
print("Confusion matrix:¥n%s" %
metrics.confusion_matrix(test_y, Test_y))
```

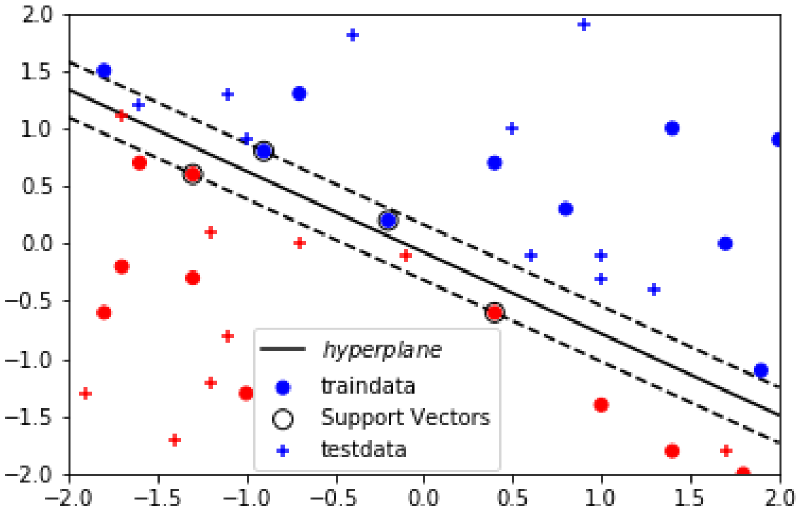
Exercise 1-Linear model(2)

```
#plot the training data
label=train_y.copy()
label[label<0]=0
label=label.astype(int)
label=label.values
colormap=np.array(['r','b'])
plt.scatter(train_x.iloc[:,0], train_x.iloc[:,1],
zorder=3, marker='o', c=colormap[label],
label='traindata')

#plot the support vectors
plt.scatter(clf.support_vectors_[:,0],
clf.support_vectors_[:,1],zorder=2,facecolors
='none', s=80, edgecolors='k', label='Support
Vectors')
```

```
#plot the hyperplane
w=clf.coef_[0]
xx=np.linspace(-2, 2)
yy=-(w[0]*xx+clf.intercept_[0])/w[1]
plt.axis([-2, 2, -2, 2])
plt.plot(xx, yy, 'k-', label='$hyperplane$')

#calculate the bias of margins
margin=1/np.sqrt(np.sum(clf.coef_**2))
yy_down=yy-np.sqrt(1+(w[0]/w[1])**2)*margin
yy_up=yy+np.sqrt(1+(w[0]/w[1])**2)*margin
#plot margins
plt.plot(xx, yy_down, 'k--')
plt.plot(xx, yy_up, 'k--')
```

Exercise 1-Linear model(3)

```
#plot the test data set
labelt=test_y.copy()
labelt[labelt<0]=0
labelt=labelt.astype(int)
labelt=labelt.values
plt.scatter(test_x.iloc[:,0], test_x.iloc[:,1], zorder=3, marker='+',
c=colormap[labelt], label='testdata')

plt.legend(loc=[0.26,0.01])
plt.savefig(path+'¥¥svc-linear.png')
plt.show()
```



Exercise 2-nonlinear model-Tasks

- First load the training data and testing data of a nonlinear example
- Create a SVM by SVC with three kernels
- Train the SVM model by the data in training file
- Classify the data in test file
- Plot the figure of data points and the hyperplane
- Pls change the parameter
    - Change parameter C and observe
    - Change parameter γ and observe

Exercise 2-nonlinear model(1)

```
import numpy as np
import pandas as pd
from sklearn.svm import SVC
import matplotlib.pyplot as plt
import os

# load data
path=os.getcwd()
traindata=pd.read_csv(path+'¥¥traindata.csv')
train_x=traindata.iloc[:, :-1]
train_y=traindata.iloc[:, -1]
testdata=pd.read_csv(path+'¥¥testdata.csv')
test_x=testdata.iloc[:, :-1]
test_y=testdata.iloc[:, -1]
```

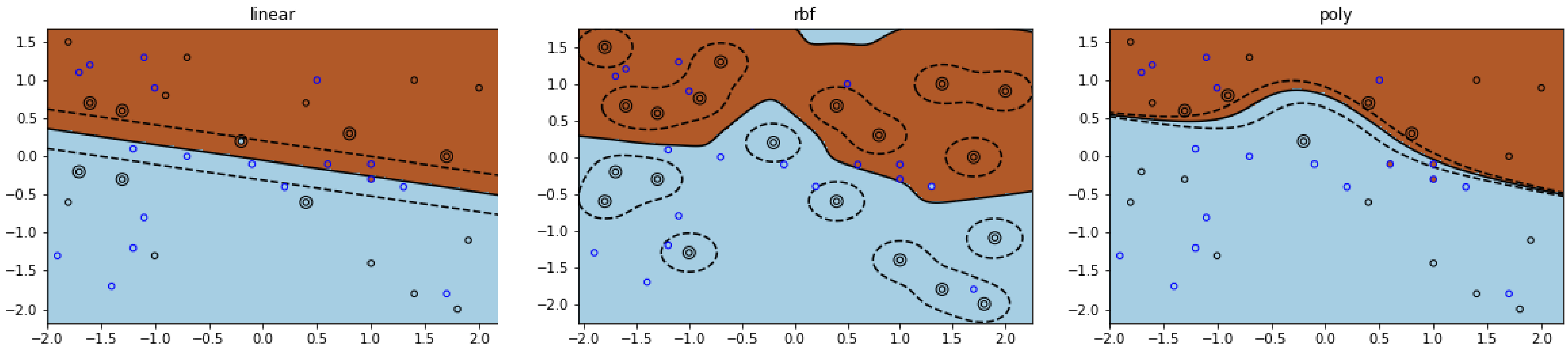
```
# introduce the SVC and fit the model
for fig_n, kernel in enumerate(('linear', 'rbf', 'poly')):
    clf=SVC(C=1.0, kernel=kernel, gamma=10)
    clf.fit(train_x, train_y)

    print('Classification report: %s¥nAccuracy rate:%s¥n'
      % (clf, clf.score(test_x, test_y)))

    #plot new window for figure
    plt.figure(fig_n)
    #clear the current figure
    plt.clf()
```

Exercise 2-nonlinear model(2)

```
#plot the train data
plt.scatter(train_x.iloc[:,0], train_x.iloc[:,1], c=train_y.iloc[:], cmap=plt.cm.Paired,
            edgecolor='k', zorder=10, s=20)

#plot the support vectors
plt.scatter(clf.support_vectors_[:,0], clf.support_vectors_[:,1], s=80,
            facecolors='none', zorder=10, edgecolors='k')
plt.axis('tight')
x_min, x_max = train_x.iloc[:,0].min()-1, train_x.iloc[:,0].max()+1
y_min, y_max= train_x.iloc[:,1].min()-1, train_x.iloc[:,1].max()+1

# create a mesh to plot in
XX, YY = np.mgrid[x_min:x_max:200j, y_min:y_max:200j]
Z = clf.decision_function(np.c_[XX.ravel(), YY.ravel()])
```

Exercise 2-nonlinear model(3)

```
    # Put the result into a color plot
    Z = Z.reshape(XX.shape)
    plt.pcolormesh(XX, YY, Z > 0, cmap=plt.cm.Paired)
    plt.contour(XX, YY, Z, colors=['k', 'k', 'k'],linestyles=['--', '-', '--'], levels=[-.5, 0, .5])
    #plot the test data set
    plt.scatter(test_x.iloc[:,0], test_x.iloc[:,1],c=test_y.iloc[:], cmap=plt.cm.Paired,
            edgecolor='b', zorder=10, s=20)
    plt.title(kernel)
plt.show()
```



Exercise 3-Multiclass classification-Tasks

- Pls import the digital dataset, divide the data into 5 parts
- Use 4 parts for training and others for prediction
- Tuning the parameters via cross validation
- Split the data to train and test subset
- Pls plot the first 4 images of training set
- Train the model of SVC by training data
- Print the classifier report and the score
- Plot the other 4 sub-figures in the end of prediction set

Exercise 3-Multiclass classification (1)

```
# Standard scientific Python imports
import matplotlib.pyplot as plt
import numpy as np
# Import datasets, classifiers and cross validation
from sklearn import datasets, svm
from sklearn.model_selection import cross_val_score

# The digits dataset
digits = datasets.load_digits()
print (digits.keys())
data=digits.data
target=digits.target
image=digits.images
print (data.shape)
```

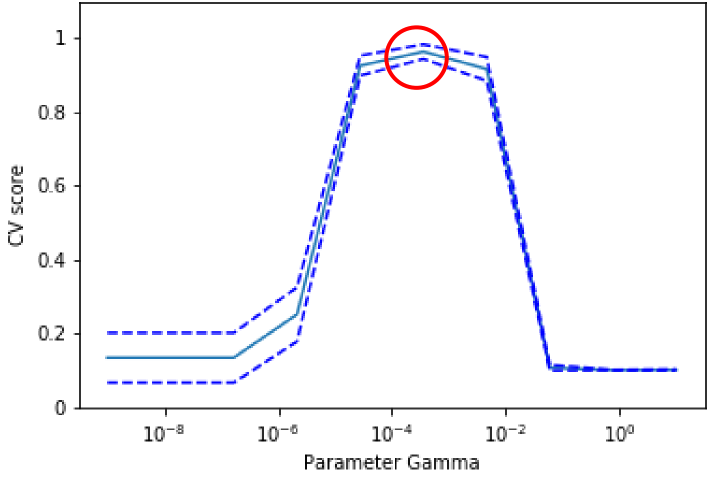
The multiclass support is handled according to a one-vs-one scheme

Exercise 3-Multiclass classification(2)

```
# define the SVC and set its parameter
clf=svm.SVC(kernel='rbf')
gamma=np.logspace(-9,1,10)

# Calculate the Cross Validation scores for clf model to different gamma
s_mean=[]
s_std=[]
for x in gamma:
    clf.gamma=x
    scores = cross_val_score(clf, data, target, cv=5)
    s_mean.append(scores.mean())
    s_std.append(scores.std())

print (s_mean)
print (s_std)
```

Exercise 3-Multiclass classification(3)

```
# plot the figure to find the best setting for gamma
plt.figure(1, figsize=(6, 4))
plt.clf()
plt.semilogx(gamma, s_mean)
plt.semilogx(gamma, np.array(s_mean) + np.array(s_std), 'b--')
plt.semilogx(gamma, np.array(s_mean) - np.array(s_std), 'b--')
locs, labels = plt.yticks()
plt.yticks(locs, list(map(lambda x: "%g" % x, locs)))
plt.ylabel('CV score')
plt.xlabel('Parameter Gamma')
plt.ylim(0, 1.1)
plt.show()

#gamma=0.001 can get the best performance
```



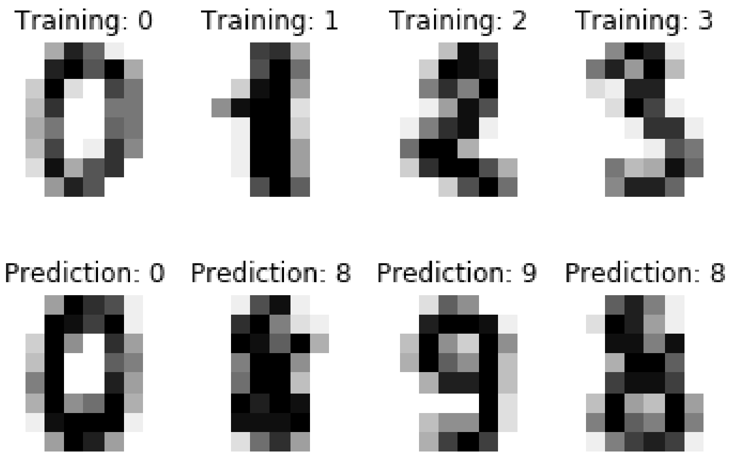
Exercise 3-Multiclass classification(4)

```
# plot the first 4 images of training set
for index in range(4):
    plt.subplot(2, 4, index + 1)
    plt.axis('off')
    plt.imshow(image[index], cmap=plt.cm.gray_r, interpolation='nearest')
    plt.title('Training: %i' % target[index])

# split arrays into train and test subsets
from sklearn.model_selection import train_test_split as split
train_x, test_x, train_y, test_y=split(data, target, test_size=0.25, shuffle=False,
random_state=0)

clf = svm.SVC(gamma=0.001)
clf.fit(train_x,train_y)
print("Classification report for classifier: %s¥nAccuracy: %s¥n"
      % (clf, clf.score(test_x,test_y)))
```

Exercise 3-Multiclass classification (5)

```
for index in range(4):
    plt.subplot(2, 4, index + 5)
    plt.axis('off')
    plt.imshow(digits.images[index-4], cmap=plt.cm.gray_r, interpolation='nearest')
    plt.title('Prediction: %i' % clf.predict(test_x)[index-4])

plt.show()
```

#subplot(numRows, numCols, plotNum)



Backup slices

sklearn.svm.NuSVC

- Nu-Support Vector Classification:
  - Similar to SVC but uses a parameter to control the number of support vectors
  - The implementation is based on libsvm
  - Parameter: nu--An upper bound on the fraction of training errors and a lower bound of the fraction of support vectors. Should be in the interval (0, 1]

```
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [1, 1], [2, 1]])
>>> y = np.array([1, 1, 2, 2])
>>> from sklearn.svm import NuSVC
>>> clf = NuSVC()
>>> clf.fit(X, y)
>>> print(clf.predict([[-0.8, -1]]))
```

sklearn.svm.LinearSVC

- Nu-Support Vector Classification:
  - Similar to SVC with parameter kernel='linear'
  - implemented in terms of liblinear rather than libsvm
  - it has more flexibility in the choice of penalties and loss functions and should scale better to large numbers of samples
- Parameters:
  - penalty: Specifies the norm used in the penalization;
  - loss: Specifies the loss function;
  - dual: Select the algorithm to either solve the dual or primal optimization problem. Prefer dual=False when n_samples > n_features.